QUESTIONS AND DISCUSSIONS.

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DISCUSSIONS.

I. Some Reflections on the Teaching of Mechanics, suggested by Professor E. V. Huntington's Article "A Logical Skeleton of Elementary Dynamics."[1]

By Alexander Ziwet, University of Michigan.

1. Even those who do not agree with Professor Huntington in all particulars, or even in his main idea, will readily admit that his article is a model of really serviceable pedagogical discussion. We have here a definite concrete scheme, carefully set out in its essential features, unassailable from the scientific point of view, and based on ripe experience in teaching. The question however remains whether Professor Huntington's "skeleton" is the only admissible scheme for teaching mechanics, and if not, whether a better scheme can be devised.

2. There is certainly something tempting and alluring in what seems to be our author's main contention that to *force* as the active, aggressive, not to say vital, principle we should concede a "logical priority" (whatever that may mean) over "inert," "dead," "passive" matter or *mass*. But we can hardly think of force otherwise than exerted by matter on matter, by one body on another body (dead or alive). As this point has been very ably discussed by Professor Hoskins it is here unnecessary to say more about it. It may suffice to say that in the "principle of force and acceleration" (p. 4) the idea of mass, here called particle, is presupposed.

I wish to state explicitly that I do not object in the least to the statement and explanation of the fundamental principles of dynamics as given in section II (pp. 3–5). The question of the "logical priority of mass over force," or vice versa, appears to me of slight importance, except for the fact that mass is a scalar and in so far more simple than the vector force.

3. This brings me to the main objection that I have to the whole trend of Professor Huntington's article: I believe that the vector idea should be emphasized more strongly, and the fundamental theorems should be stated for three (not two) dimensions. It is true that in sections VI and VII three dimensions are used. But right after this, in section VIII (top of p. 14), we find the startling statement that "any set of forces acting on a rigid body can be 'boiled down' either to a single force, or else to a single couple." Of course, the author had in mind the case of forces in a plane; but then a "rigid body" in a plane is, without further explanation, a rather artificial thing.

The use of vectors simplifies the case of three dimensions very essentially. But this is not the only advantage. Professor Huntington complains of the difficulty that students find in the notion of acceleration and proposes a remedy

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[1] See American Mathematical Monthly, Vol. 24, pp. 1–16.

(p. 2). But, however ingenious and striking the "snap shot" illustration, does it hit the vital point of the difficulty?—That the velocity in rectilinear motion may vary (and that is all the snap shots show) is hardly an unfamiliar notion to a junior in college who has studied the calculus. The difficulty arises only in curvilinear motion where velocity, and hence also acceleration, must be regarded as a vector.

In the first paragraph of section IV, where this question is discussed, it would have been far better not to mention any axes of reference which "remain fixed throughout the discussion." The formulae for tangential and normal acceleration are quite independent of any fixed axes of reference.

In sections VI and VII the misleading statements "Work = Force $\times$ Distance" and "Impulse = Force $\times$ Time," which are as mischievous as statements like "Velocity = Space/Time," etc., could be made correct by using vector notions.

4. Professor Huntington seems to base his main argument in favor of the "logical priority" of force over mass on the tables of dimensions and units on pp. 15, 16. I must confess that the inspection of these tables does not convince me at all. Such tables are a nuisance, anyhow. They may have a place in an encyclopedic handbook; I do not like to see them in a textbook; the student should not be encouraged to consult them.

But if we must have such tables, they should be arranged less artificially. The units of space and time should certainly come first, to be followed by those of velocity and acceleration. Then should come either mass and force or force and mass; and these can then both be used in defining momentum, kinetic energy, etc. It appears just as odd to me to say that momentum is force $\times$ time (by p. 15) as that impulse is mass $\times$ velocity (p. 16), and similarly for work and energy.

5. In the present state of science it would seem best, indeed necessary, to tell the student that both systems are in use, and that both are equally justifiable. It is a matter of personal opinion whether force or mass is regarded as more simple or more fundamental. Why not introduce both these notions from the beginning and use both? Is there really anything gained by always writing w/g instead of m? Why should we carefully avoid the use of the simple term mass after it has been introduced, and the simple symbol m, and use the more complicated symbol w/g and any number of terms such as matter, lump of matter, body, particle, and especially inertia (which is liable to suggest the objectionable force of inertia)?

When a past-master in the art of devising sets of axioms takes the trouble to give an exposition of the fundamental dynamic concepts and theorems, we are surely greatly indebted to him; and the work cannot fail to be clarifying and stimulating. But it is certainly only fair to the student to let him know that the system of units here advocated is not used by a single writer on higher mechanics.